TOPOLOGY

Qualifying Review Exam May 14, 1999

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1. Prove: *A closed map is a quotient map.*

2. Prove: *Let X be a compact space, $\mathcal{U}$ be an open covering of X. Then there exists a partition of unity dominated by $\mathcal{U}$.*

3. Let (X, d) be a metric space.
P rove: *If X is complete and totally bounded (for every $\epsilon > 0$, there exists a finite covering of X by ϵ-balls), then X is sequentially compact.*
G ive an example showing *totally bounded* in the statement (1) cannot be replaced by *bounded*.

4. Using only the definition of "compactness", prove:

The closed interval $I = [0, 1] \subset \mathrm{R}$ *is compact.*

5. A subspace S of X is called a **retract** of X if there exists a continuous map $r : X \to S$ such that $r \circ i$ is homotopic to 1_S. Prove: *The equator S^1 of the sphere S^2 is not a retract of S^2.*

6. Let T be the torus, M be a closed disk. Form a connected sum $X = T\#M$. [That is, remove open disks $D_1 \subset T$ and $D_2 \subset M$, and glue a cylinder $S^1 \times I$ by homeomorphisms $S^1 \times \{0\} \xrightarrow{\cong} \partial D_1$ and $S^1 \times \{1\} \xrightarrow{\cong} \partial D_2$, where ∂ means the boundary]. Describe X homotopically, and calculate the fundamental group of X.