

SYLLABUS
ALGEBRA QUALIFYING EXAMINATION
Spring 2003

1. Groups.

Subgroups, cosets, normality, quotient groups, centralizers, normalizers, Lagrange's theorem, homomorphism theorems, automorphisms, direct and semi-direct products, groups actions, orbits, isotropy, transitive actions, counting, Sylow theorems, free groups, generators and relations, classification of finitely generated abelian groups, composition series, solvability

Lang: Chapter I, §1–8, 12

2. Rings and Modules.

Ideals, subrings and submodules, quotient rings and quotient modules, algebras over commutative rings, homomorphism theorems, products, sums and composition series of modules, prime and maximal ideals, integral domains, polynomial rings, factorization, free modules, classification of finitely generated modules over principal ideal rings, Euclidean rings, rings of fractions and localization

Lang: Chapter II (all), Chapter III §1–7; Chapter IV, §1–3.

3. Fields.

Algebraic and transcendental extensions, algebraic closure, splitting fields, normality and separability, Galois extensions, main theorem of Galois theory, Galois group of a polynomial, solvability by radicals

Lang: Chapter V (all except §6), Chapter VI, §1–3,7, Chapter VIII §1

4. Linear Algebra.

Vector spaces, linear independence and spans, basis and dimension, linear transformations, characteristic and minimal polynomials, eigenspaces, diagonalizability, similarity, Jordan and rational canonical forms

Lang: Chapter XIII §1-5, Chapter XIV (all)

Reference:

S. Lang, *Algebra, Third Edition*, Addison-Wesley 1993