Detailed Syllabus for 2003 Qualifying Exam in Topology

References.

- *Topology, 2nd ed.* , by J. R. Munkres. Prentice Hall (2000).
- Chapter 1 of *Algebraic Topology*, by A. Hatcher.
- *Algebraic Topology: an introduction*, by W. S. Massey. Springer GTM #56.

1. Basics
   (a) Topology on a set, basis, subbasis
   (b) Continuity, homeomorphism, using maps to induce topologies on new spaces
   (c) Subspace topology, product topology, quotient topology
   (d) Closure, interior, limit points, convergence of sequences
   (e) Metric spaces, metric topology
2. Connectedness
   (a) Connectedness, and behavior under continuous maps, unions, products etc
   (b) Local connectedness, components
   (c) Path connectedness, path components
   (d) Connectedness in linear continua, IVT
3. Compactness
   (a) Compactness, open covers, continuous maps, subspaces, etc
   (b) Compact metric spaces, Lebesgue numbers, sequential compactness
   (c) Compactness in $\mathbb{R}^n$, EVT, Uniform continuity theorem.
   (d) Limit point compactness
   (e) Local compactness, one point compactifications
   (f) (Wright's proof of the) Tychonoff Theorem
4. Separation and Countability Axioms, Basic Metrization Results
   (a) First and second countable spaces, Lindeloff spaces
   (b) Hausdorff, regular, normal
   (c) Urysohn lemma, Teitse extension theorem
   (d) Urysohn metrization, embedding compact manifolds in $\mathbb{R}^n$, partitions of unity
5. Homotopy and the Fundamental Group
   (a) Homotopies, homotopies of paths, fundamental group, induced homomorphisms
   (b) π_1 is a functor
   (c) Fundamental group of the circle (via covering spaces)
   (d) Homotopy type, retracts, deformation retracts

6. Preliminary Applications

   (a) Brouwer fixed point and no retraction theorems
   (b) Fundamental theorem of algebra
   (c) Borsuk-Ulam, and applications (Ham sandwich theorem)
   (d) $\pi_1(S^n)$
   (e) Fundamental group of a union of two spaces when the intersection has more than one path component
   (f) Jordan separation and Jordan curve theorems
   (g) Embedding graphs in the plane
   (h) Winding numbers

7. Some Group Theory

   (a) Free products of groups, existence and universal property, uniqueness
   (b) Reduced word description of elements, centralizers, centers and torsion elements in free products, free groups
   (c) Commutators, commutator subgroups, abelianizations, free abelian groups
   (d) Finitely generated and finitely presented groups, a presentation of a group
   (e) Teitze transformations, manipulating presentations (converting between the Wirtinger and $a^p = b^q$ presentations of a (p, q)-torus knot complement group).

8. Seifert-van Kampen Theorem and applications

   (a) Seifert-van Kampen theorem, statement and sketch of proof
   (b) Applications of S-vK: fundamental groups of graphs, 2-complexes, n-complexes.
   (c) The effect of attaching a 1-cell, a 2-cell, an n-cell ($n > 2$)
   (d) Presentation 2-complexes, Cayley complexes, Cayley graphs
   (e) (p, q)-torus knots, and their groups
   (f) The Wirtinger presentation for general link complements
   (g) The Conway Zip proof of the classification of closed surfaces, using π_1 to distinguish between closed surfaces

9. Covering Spaces

   (a) The homotopy lifting property: HLP, and its consequences; the path lifting property and the path-homotopy lifting property.
   (b) General lifting criterion. Uniqueness of lifts.
   (c) Existence of universal covering spaces.
   (d) The Galois correspondence between based covers of X and subgroups of $\pi_1(X)$.
   (e) Regular covering spaces and deck transformations. Group actions.
   (f) Examples: Cayley complexes, manifold examples.
   (g) Permutation representations and covering spaces. Examples and applications to branched covers.