Syllabus for 2004 Qualifying Exam in Topology

1. Fundamentals
   a. topology on a set, basis and subbasis for a topology
   b. subspace topology, product topology, quotient topology
   c. continuity, uniform continuity, homeomorphisms, quotient maps
   d. closure, interior, limit points, convergence of sequences
   e. metric spaces, metrization theorems
2. Connectedness
   a. connectedness, local connectedness, components
   b. path-connectedness, local path-connectedness, path components
   c. preservation of connectedness under continuous maps, unions, products, etc.
3. Countability and Separation Axioms
   a. first countable, second countable, separable
   b. Hausdorff, regular, completely regular, normal
   c. Urysohn's Lemma, Tietze Extension Theorem
4. Compactness
   a. compactness, local compactness, compactification
   b. limit point compactness, sequential compactness, Lebesgue numbers
   c. preservation of compactness under continuous maps, unions, products, etc.
   d. Tychonoff's Theorem
5. Complete Metric Spaces, Function Spaces
   a. completeness, completion, total boundedness
   b. Baire spaces
   c. compact-open topology on function spaces
   d. evaluation map, exponential correspondence
6. Fundamental Groups and Covering Space Theory
   a. homotopy of maps, homotopy of paths, fundamental group, Van Kampen's Theorem
   b. homotopy equivalence, contractibility, deformation retraction
   c. covering spaces, path lifting, regular covering spaces, subgroup correspondence theorem
7. Examples
   a. S_Ω, $\overline{S}_\Omega$
   b. lower limit topology, cofinite topologies, Cantor sets
   c. countable and uncountable products of $[0,1]$, $\mathbb{R}$
   d. manifolds, classification of surfaces

References:

Topology: A First Course by J. Munkres
Algebraic Topology: An Introduction by Wm. Massey
Chapter 1 of *Algebraic Topology* by Allen Hatcher, available at
`http://math.cornell.edu/~hatcher/#ATI`