Real Analysis **Math 5453/63** **2004/2005**

Syllabus for Qualifying Examination May/August 2005

Text: *Real Analysis, 3rd ed. by H. L. Royden*

Topics: Algebras of Sets, Real Number System, Lebesgue measure, Lebesgue Integral, Differentiation and Integration, L^p - Spaces on $\mathbb{R}$, General Measure and Integration, Construction of measures, Invariant measures or weak derivatives and Sobolevspaces..

Important Issues:

Theory of functions of one variable: Generation of σ-algebras, Completeness Axioms of $\mathbb{R}$, Cantor sets. Topology of $\mathbb{R}$ and continuous functions; measurable functions, a.e convergence, convergence in measure uniform convergence. Integral of nonnegative measurable functions and integrable functions. Convergence Theorems of Fatou, B. Levi, and Lebesgue.

Functions of bounded variation and absolutely continuous functions, Hölder and Minkowski inequalities, Theorem of Fischer - Riesz and Riesz representation theorem.

General Measure and Integral: Measure spaces, measure, complete and σ -finite measures. Measurable functions and limits of measurable functions.

Simple functions and Integration, convergence Theorems. Signed measures and decomposition of measures. Theorem of Radon and Nikodym. L^p-spaces for general measure spaces, Hölder and Minkowski inequalities, Theorem of Fischer - Riesz and Riesz representation theorem.

Carathéodory's construction of measures: Set functions on a semi algebra $\rightarrow$ measures on an algebra $\rightarrow$ outer measure $\rightarrow$ measure. Lebesque-Stieljes Integral, Product measures, Theorems of Fubini and Tonelli, Hausdorff measure.

a) Homogeneous spaces, topologically equicontinuous measures, existence of invariant measures, (Haar measures).
b) Definition of weak derivatives, Sobolev space, Sobolev Imbedding theorem (for $p < n$).

Note: There will be a choice between problems to a) and b).