Syllabus for 2005 Qualifying Examinations in Topology

The Fall, 2004 and Spring, 2005 graduate topology courses are based on the book *Topology: A Geometric Approach,* by Terry Lawson. Since the text was followed rather loosely, however, the best guide to the qualifying examinations is the material available on the course websites:

`http://www.math.ou.edu/~dmccullough/teaching/f04-5853/`
`http://www.math.ou.edu/~dmccullough/teaching/s05-5863/`

This material includes homework problems, solutions to many of the homework problems, and the exams, for which complete solutions are posted.

I. Topologies
   1. Metric spaces, the metric topology
   2. Definition of a topology, key examples such as the discrete topology, the standard topology on $\mathbb{R}^n$, the cofinite topology, the lower limit topology on $\mathbb{R}^n$
   3. Basis for a topology, recognition of bases, second countable spaces
   4. Subspace topology
   5. Closed sets, closure and interior, limit points
   6. Separation properties: Hausdorff, regular, normal
   7. Sequences and convergence, dense subsets, separable spaces

II. Continuous maps
   1. Definitions of continuity, continuity and convergent sequences
   2. Piecing together continuous maps on open covers and locally finite closed covers
   3. Homeomorphisms
   4. Isometries of the plane
   5. Barycentric coordinates and affine homeomorphisms of the plane

III. Compactness
   1. Definition and basic properties of compactness
   2. Compact subsets of $\mathbb{R}^n$
   3. Properties equivalent to compactness in metrizable spaces
   4. Lebesgue numbers
   5. The Extreme Value Theorem
   6. Compactification
   7. Local compactness, the one-point compactification, stereographic projection of $\mathbb{R}^n$
   8. The Tychonoff Theorem, statement and applications (not the proof)

IV. Product topology
   1. Definition of the product topology
   2. Continuous maps into products
   3. The product topology for infinite products

V. Connectedness
   1. Definition and basic properties of connectedness
   2. The Intermediate Value Theorem
   3. Path-connectedness and path components
   4. Local path-connectedness

VI. Urysohn's Lemma and the Tietze Extension Theorem
   1. Statements of Urysohn's Lemma and the Tietze Extension Theorem
   2. Uniform convergence of sequences of continuous functions

VII. Quotient topology
   1. Definition of the quotient topology
   2. The universal mapping property of quotients
   3. Identification spaces

VIII. Manifolds
   1. Definition of manifold, boundary of a manifold, examples of manifolds
   2. The Collar Theorem, Invariance of Domain
   3. Handles, handle decompositions of surfaces
   4. Homotopy and isotopy, sliding handles
   5. Classification of 2-manifolds up to homeomorphism
   6. Orientable and nonorientable 2-manifolds, Euler characteristic

IX. Fundamental groups
   1. Definition of $\pi_1(X, *)$, change of basepoints
   2. The covering map $p\colon \mathbb{R} \to S^1$ and the fundamental group of the circle
   3. Simply-connected spaces, S^n is simply-connected for $n \geq 2$
   4. Basic applications
   5. Homotopy equivalences

X. Function spaces (a reference for this is *Topology: A First Course,* by James Munkres)
   1. Definition and basic properties of the compact-open topology

XI. Covering spaces
   1. Definition of covering spaces, basic examples
   2. The Lifting Criterion
   3. Covering transformations, regular coverings and normal subgroups
   4. The universal covering space
   5. The Galois correspondence between covering spaces of X and subgroups of $\pi_1(X, *)$