Syllabus for Topology Qualifying Exam, 2006

The 2005–2006 topology graduate course used the books *Topology* (second edition) by Munkres and *Algebraic Topology* by Hatcher (chapters 0 and 1). The book *Algebraic Topology: An Introduction* by Massey is also recommended, as it provides more detail than Hatcher in some areas (some of these details found their way into the course lectures).

General Topology:

I. Topological spaces and continuous maps (Munkres, sections 12–20, 22)
1. Topological spaces, bases, subbases
2. The order topology
3. The product topology (two factors)
4. The subspace topology
5. Closed sets and limit points, Hausdorff spaces
6. Continuous maps, homeomorphisms, local continuity, pasting lemma, maps into products
7. The product topology (general case), box topology
8. Metric spaces, uniform topology
9. The quotient topology, maps out of quotient spaces

II. Connectedness and compactness (Munkres, sections 23–27, 29)
1. Connected spaces, connectedness of products
2. Connectedness in linear continua, intermediate value theorem, path connectedness
3. Components and local connectedness
4. Compact spaces: continuous maps, products, tube lemma, finite intersection property
5. Extreme value theorem, Lebesgue number lemma
6. Local compactness, one-point compactification

III. Countability and separation axioms (Munkres, sections 30–32)
1. First and second countability axioms
2. regular spaces, normal spaces

IV. Urysohn's Lemma and applications (Munkres, sections 33, 35, 36)
1. Urysohn's Lemma, separation by continuous functions
2. Tietze extension theorem
3. Partitions of unity
4. Embeddings of manifolds

V. Other topics (Munkres, sections 37, 46)
1. The Tychonoff theorem (statement and applications only)
2. The compact-open topology, the evaluation map, induced maps

Algebraic Topology:

VI. The fundamental group (Munkres, sections 51–52, 54–55, 57–60; Hatcher, section 1.1)
1. Path homotopy, groupoid properties
2. Fundamental group, induced homomorphisms
3. Fundamental group of the circle (via a covering space)
4. Retractions and the fundamental group, Brouwer fixed point theorem
5. The Borsuk-Ulam theorem, applications
6. Deformation retractions, homotopy equivalences

7. Fundamental group of the sphere/easy van Kampen theorem (simply connected pieces)

VII. Covering spaces (Hatcher, section 1.3; Munkres, sections 53–54, 79–82)

1. Definition of covering spaces
2. Path lifting and uniqueness
3. Injectivity of induced map on fundamenal group
4. The lifting criterion, uniqueness of lifts
5. Existence of universal covering spaces (statement and applications only)
6. Galois correspondence between subgroups and covering spaces (see also Massey, chapter 5, sections 3–6) (statement and applications only)
7. Covering translations (or deck transformations/automorphisms)

VIII. The van Kampen theorem (Hatcher, section 1.2 and chap. 0; Munkres, sections 69–73)

1. Free products of groups, existence, the mapping property
2. The van Kampen theorem (statement and applications only)
3. 1- and 2-dimensional CW complexes, attaching cells, collapsing a contractible subcomplex
4. Generators and relations

IX. Graphs and free groups (Hatcher, section 1.A; Massey, chapter 6)

1. Graphs = 1-dimensional CW complexes
2. Contractibility of trees
3. Fundamental groups of graphs are free groups
4. Subgroups of free groups are free
5. Finding free generators of a subgroup of a free group, via covering spaces of graphs
6. Euler characteristic of a graph