Main results

Real Analysis I and II, MATH 5453-5463, 2006-2007

Section		Homework
	Introduction.	
1.3	Operations with sets. DeMorgan Laws.	
1.4	Proposition 1. Existence of the smallest algebra containing C.	
2.5	Open and closed sets.	
2.6	Continuous functions. Proposition 18.	Hw #1. p.16 #9, 11, 17, 18; p.19 #19.
2.7	Borel sets.	p.49 #40, 42, 43; p.53 #53*.
3.2	Outer measure.	
	Proposition 1. Outer measure of an interval.	
	Proposition 2. Subadditivity of the outer measure.	
	Proposition 5. Approximation by open sets.	
3.3	Measurable sets.	
	Lemma 6. Measurability of sets of outer measure zero.	
	Lemma 7. Measurability of the union.	Hw #2. p.55 #1-4; p.58 # 7, 8.
	Theorem 10. Measurable sets form a sigma-algebra.	
	Lemma 11. Interval is measurable.	
	Theorem 12. Borel sets are measurable.	
	Proposition 13. Sigma additivity of the measure.	
	Proposition 14. Continuity of the measure.	
	Proposition 15. Approximation by open and closed sets.	Hw #3. p.64 #9-11, 13, 14.
3.4	A nonmeasurable set.	
3.5	Measurable functions.	
	Proposition 18. Equivalent definitions of measurability.	
	Proposition 19. Sums and products of measurable functions.	
	Theorem 20. Infima and suprema of measurable functions.	Hw #4. p.70 #18-22.
3.6	Littlewood's three principles.	
	Egoroff's theorem.	
	Lusin's theorem.	
4.2	Prop.2. Lebesgue's integral of a simple function and its props.	
	Lebesgue's integral of a bounded measurable function.	
	Proposition 3. Criterion of integrability.	
	Proposition 5. Properties of integrals of bounded functions.	
	Proposition 6. Bounded convergence theorem.	
4.3	Lebesgue integral of a nonnegative function and its properties.	
	Theorem 9. Fatou's Lemma.	
	Theorem 10. Monotone Convergence Theorem.	
	Proposition 14. Continuity of the integral.	Hw #5. p.89 #3-4, 6-7, 9.
4.4	Lebesgue integral of a general function.	

Proposition 15. Properties of integrals of general functions.
Theorem 16. Lebesgue Dominated Convergence Theorem. — Hw #6. p.93 #10(a), 11-15.

5.1 Derivates of a function.
Vitali Lemma (no proof).
Theorem 3. Differentiation of a monotone function. — Hw #7. p.101 #1-5.

5.2 Functions of bounded variation.
Lemma 4. Properties of the variations.
Theorem 5. A BV function is the difference of increasing funcs. — Hw #8. p.104 #7-11.

5.3 Differentiation of an integral.
Lemma 7. Indefinite integral is continuous and BV.
Lemma 8. Indefinite integral is 0 implies f=0.
Theorem 10. (includes Lemma 9) G'(x)=f(x) a.e.

5.4 Absolutely continuous functions.
Lemma 11. AC function is BV.
Lemma 13. If f is AC, and f'(x)=0 a.e, then f=const.
Theorem 14. f is an indefinite integral if and only if it is AC. — Hw #9. p.110 #12, 14, 15, 18, 20(a,b).

5.5 Convex functions.
Lemma 16. Secant Lemma.
Propositions 18-19. f is convex iff f">=0.
Proposition 20. Jensen Inequality. — Hw #10. p.116 #23(a,b), 25-28.

6.1 Lp spaces.
6.2 Theorem 1. Minkowski Inequality.
Theorem 4. Holder Inequality. — Hw #11. p.119 #1-4; p.122 #7.

6.3 Convergence and completeness.
Proposition 5. Criterion of completeness.
Theorem 6. Riesz-Fischer Theorem. — Hw #12. p.126 #9-15.

6.4 Approximation in Lp.
Lemma 7. Approximation by bounded functions.
Proposition 8. Approximation by step and continuous functions.

S.1 Hilbert spaces.
Cauchy-Schwarz inequality.
Theorem. Closed unit ball in H is not compact.
Theorem. Linear functional is continuous iff it is bounded.
Projection Theorem.
Riesz Representation Theorem for Hilbert spaces.

6.5 Proposition 11. g in Lq defines a bounded functional on Lp.
Theorem 13. Riesz Representation Theorem (no proof).

7.1-7.2 Metric spaces. Continuous functions. Compact sets.
Uniform limit of continuous functions.

	Weierstrass M-test for continuous functions.	
	First Dini's Theorem.	
	Urysohn's lemma.	Hw #1. Chap. 7 #1, 4, 7-8, 11(a), 12-16.
	Tietze's Extension Theorem.	
9.9	Second Dini's Theorem.	
	Kakutani-Krein Theorem.	
	Stone-Weierstrass theorem	Hw #2. p. 213 #42-46; S. 1.1, 1.2.
11.1	Nonnegative measure.	
	Propositions 1-4. Properties of a measure.	
11.2	Propositions 5-8. Properties of measurable functions.	Hw #3. Chap. 11 #1, 3-5, 7, 10-13.
11.3	Integration.	
	Theorem 11. Fatou's lemma.	
	Theorem 12. Monotone Convergence Theorem.	
	Propositions 13-15. Properties of integrals.	
	Theorem 16. Lebesgue's Dominated Convergence Theorem.	Hw #4. p.267 #17, 19-22.
11.5	Signed measures.	
	Propositions 19-20. Positive sets.	
	Proposition 21. Hahn Decomposition Theorem.	
	Proposition 22. Jordan Decomposition Theorem.	Hw #5. p. 275 #27(a), 29-31.
11.6	Lemma 9. Measurable function on nested sets.	
	Theorem 23. Radon-Nikodym Theorem.	
	Proposition 24. Lebesgue Decomposition Theorem.	Hw #6. p. 279 #33(a)-37.
11.7	Lemma. g in Lq defines a continuous linear functional on Lp.	
	Lemma 27. Criterion for g to be in Lq.	
	Theorem 29. Riesz Representation Theorem for Lp.	
12.1	Outer measure.	
	Theorem 1. Outer measure generates a measure (no proof).	Hw #7. S. 2.1-2.3; p. 291 # 1, 2.
12.2	Semialgebras.	
	Theorem C.1. Extension from a semialgebra.	
12.4	Product measures.	
	Lemma C.3. Measurable rectangles form a semialgebra.	
	Theorem C.4. Set function on a semialgebra.	
	Examples of functions measurable on product spaces.	
	Lemma 15. Measurability of cross-sections.	
	Lemma 16. Area of an R_σδ set.	
	Lemma 17. Sets of measure zero.	
	Proposition 18. Area by iterated integrals.	
	Theorem 20. Tonelli's Theorem.	
	Theorems 19. Fubini's Theorem.	
	Existence of the convolution f*g for integrable f and g.	Hw #8. p. 310 #19, 21, 22, 30; S. 3.1-3.3.

10.1	Banach spaces.	
10.2	Dual spaces.	
	Proposition 10.3. X* is a Banach space.	
	Theorem. C(K) is a Banach space.	
7.10	Equicontinuous sets in C(K).	Hw #9. p. 218 #1-3; p. 222 #13-14;
	Propositions 7.37-7.41. Arzela-Ascoli Theorem.	p. 169 #47, 50.
	Integral operators in C[0,1].	
	Theorem. Baire and Borel sets in metric spaces.	
	Theorem. Regularity of Borel measures on compact metric spaces.	
9.4	Proposition 9.16. Partition of unity.	
13.4-5	Theorem. Decomposition of functionals into F+ and F-.	
	Theorem. Representation of positive functionals.	
	Riesz-Markov Representation Theorem for [C(X)]*.	Hw #10. S. 4.1-4.5.
S.1	Theorem S.1.1. Approximation by simple functions.	
	Theorem S.1.2. Approximation by continuous functions.	
S.2	Theorem S.2.1. Existence of convolutions in Lp.	
	Mollifiers.	
	Theorem S.2.2. Differentiation of an integral w/r to a parameter.	
	Fourier Transform of a Gaussian.	Hw #11. S. 5.2-5.6.
	Theorem S.2.3. Regularization of functions.	
	Theorem S.2.4. Approximation by smooth functions.	