Algebra Qualifying Exam Syllabus

(Prepared by J. Kujawa, Spring 2010)

Disclaimer: The list of topics presented below is intended to be reasonably representative but is not guaranteed to be exhaustive. All of these topics are covered in the course text "Abstract Algebra: 3rd Edition" by Dummit and Foote. With some exceptions, the course during the 2009-2010 academic year covered Chapters 1-9, 12-14, and portions of Appendix II of Dummit and Foote.

1. **Group Theory**

   - Groups, subgroups, cosets, homomorphisms, Lagrange's Theorem.
   - Quotient groups and the universal property of quotient groups, the isomorphism theorems and lattice theorem for subgroups.
   - Finite groups, cyclic groups, permutation groups, dihedral groups, general linear and special linear groups over finite and infinite fields, and other standard examples.
   - Free groups, presentations, generators and relations, direct and semidirect products of groups.
   - Classification of finitely generated abelian groups.
   - Group actions, stabilizers, orbits, class equation, Cayley's Theorem.
   - Sylow theorems and proving/disproving simplicity of finite groups.
   - Composition series and the Jordan-Holder-Remak Theorem for finite groups.

2. **Ring Theory**

   - Rings, subrings, ideals, homomorphisms.
   - Polynoimal rings, group rings, matrix rings, rings of continuous/differentiable/etc. functions, and other standard examples.
   - Quotient rings and the universal property of quotient rings, the isomorphism theorems and lattice theorem.
   - Maximal ideals, prime ideals, radical and nilradical. The maximal and prime ideal spectrum of a commutative ring.
   - Integral domains, irreducible and prime elements, euclidean rings, principal ideal domains, unique factorization domains.
   - Multiplicatively closed sets, localization of rings, and the universal property of localization.
   - Field of fractions of an integral domain.
   - Factorization in polynomial rings, Gauss lemma, Eisenstein Criterion
   - Modules, free modules, direct sums and products, quotient modules, the isomorphism theorems for modules.
   - Tensor products of modules.
   - Modules over a Principal Ideal Domain, Rational Canonical Form, Jordan Canonical Form.
   - Noetherian rings and modules.

3. **Field Theory**

   - Fields, algebraic extensions, minimal polynomial, degree, algebraic closure.
   - Splitting fields, separable polynomials, primitive element theorem.
   - Transcendental elements and extensions, function fields.
   - Finite fields.
   - Normal extensions, Galois theory.

4. **Category Theory**

   - Definition of a category and many standard examples (e.g. Groups, Abelian Groups, Rings, Commutative Rings, F-Vector Spaces, finite dimensional F-vector spaces, finite directed graphs, etc.).
   - Definition of a (covariant) functor and many standard examples as they arise in covering the material listed above (e.g. creation of a group's group ring, or creation of a matrix ring with entries from a commutative ring are functorial).