

## Fundamentals

- Topology on a set, basis, subbasis
- Continuity, homeomorphism, using maps to induce topologies on new spaces
- Subspace topology, product topology, quotient topology
- Closure, interior, limit points, convergence of sequences
- Metric spaces, metric topology

## Connectedness

- Connectedness, local connectedness, components
- Path connectedness, local path connectedness, path components
- Connectedness and continuous maps, unions, products

## Compactness

- Compactness, local compactness, compactification
- limit point compactness, Lebesgue numbers, sequential compactness
- compactness and continuous maps, unions, products etc. Tychonoff Theorem

## Separation and Countability Axioms, Basic Metrization Results

- First and second countable spaces, separable
- Hausdorff, regular, normal
- Urysohn lemma, Tietze extension theorem
- Urysohn metrization, embedding compact manifolds in  $\mathbb{R}^n$ , partitions of unity

## Homotopy and the Fundamental Group

- Homotopies, homotopies of paths, fundamental group, induced homomorphisms, functoriality,
- Homotopy type, retracts, deformation retracts

- Brouwer fixed point theorem

### Van Kampen Theorem, Computations

- free groups, free products of groups, Seifert-van Kampen theorem.
- fundamental groups of surfaces (classification of surfaces)
- CW complexes
- fundamental groups of complexes
- Cayley graphs, Cayley 2-complexes, presentation 2-complexes

### Covering Spaces

- path lifting, homotopy lifting.
- General lifting criterion for covering spaces. Uniqueness of lifts.
- Existence of universal covering spaces.
- The Galois correspondence between based covers of  $X$  and subgroups of  $\pi_1(X)$ .
- Regular covering spaces and deck transformations. Group actions.
- Permutation representations and covering spaces.
- Subgroups of free groups and free products: Schreier and Kurosh Theorems