Algebra Qualifying Exam Syllabus

(Prepared by A. Pitale and J. Kujawa, Spring 2012)

Disclaimer: The list of topics presented below is intended to be reasonably representative but is not guaranteed to be exhaustive. All of these topics are covered in the course text "Abstract Algebra: 3rd Edition" by Dummit and Foote. With some exceptions, the course during the 2011-2012 academic year covered Chapters 1-10 and 13-14 of Dummit and Foote. More specifically:

1. **Group Theory**

   - Groups, subgroups, cosets, homomorphisms, Lagrange's Theorem.
   - Quotient groups, the isomorphism theorems and lattice theorem for subgroups.
   - Finite groups, cyclic groups, permutation groups, dihedral groups, general linear and special linear groups over finite and infinite fields, and other standard examples.
   - Free groups, presentations, generators and relations, direct products of groups.
   - Classification of finitely generated abelian groups.
   - Group actions, stabilizers, orbits, class equation, Cayley's Theorem.
   - Sylow theorems and proving/disproving simplicity of finite groups.
   - Composition series, Jordan-Hölder Theorem for finite groups.

2. **Ring Theory**

   - Rings, subrings, ideals, homomorphisms.
   - Polynomial rings, group rings, matrix rings, rings of continuous/differentiable/etc. functions, and other standard examples.
   - Quotient rings, the isomorphism theorems and lattice theorem.
   - Maximal ideals, prime ideals, radical and nilradical. The maximal and prime ideal spectrum of a commutative ring.
   - Integral domains, irreducible and prime elements, Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains.
   - Multiplicatively closed sets, localization of rings.
   - Field of fractions of an integral domain.
   - Factorization in polynomial rings, Gauss's lemma, Eisenstein Criterion.
   - Modules, free modules, direct sums and products, quotient modules, the isomorphism theorems for modules.
   - Tensor products of modules. Basis Theorem for tensor product of finite dimensional vector spaces.
   - Universal Property of tensor products.

3. **Field Theory**

   - Fields, algebraic extensions, minimal polynomial, degree, algebraic closure.
   - Splitting fields, separable polynomials, primitive element theorem.
   - Transcendental elements and extensions, function fields.
   - Finite fields.
   - Cyclotomic polynomials.
   - Normal extensions, Galois theory.
   - Geometric constructions and solvability of polynomials by radicals.