Real Analysis Qualifying Exam Syllabus (Aug. 2012, Jan. 2013)

Topics: Real number system, continuous functions, Lebesgue measure, Lebesgue integral, differentiation and integration of functions of one variable, general measure and integration theory, construction of measures, function spaces, invariant measures or weak derivatives and Sobolev spaces.

1. **Theory of functions of one variable:**

   - Axioms for real numbers, sequences of real numbers, Cauchy sequences, topology of $\mathbb{R}$, sequences of continuous functions, Arzela-Ascoli Theorem.
   - Algebras and σ-algebras, Borel sets, F_δs, G_δs, Cantor sets, set functions, Lebesgue's outer measure, characterization of measurable sets.
   - Measurable functions, a.e. convergence, convergence in measure, uniform convergence, approximation of measurable functions with simple functions and continuous functions, Theorems of Egorov and Lusin.
   - Lebesgue integral on $\mathbb{R}$, integral of nonnegative measurable functions and integrable functions, convergence theorems (Fatou, B. Levi, Lebesgue), equi-integrable (uniformly integrable) functions.
   - Vitali cover, derivatives, functions of bounded variation and absolutely continuous functions, Cantor-Lebesgue function, derivative of an indefinite integral.

2. **General Measure and Integral:**

   - Measure spaces, complete and σ-finite measures, measurable functions, limits of measurable functions, Egorov's Theorem,
   - Integral of nonnegative measurable functions, integrable functions, convergence theorems (Fatou, B. Levi, Lebesgue), equi-integrable (uniformly integrable) functions.
   - Signed measures, absolutely continuous and singular measures, decomposition of measures, Radon-Nikodym Theorem, Riesz Representation Theorem.
   - Constructions of measure, set functions on a semi-algebra, measures on an algebra, outer measure, σ-algebra of μ^*-measurable sets.
   - Baire measure, Lebesgue-Stieltjes integral, product measures, Theorems of Fubini and Tonelli.
   - Caratheodory outer measure, sets separated by functions, Hausdorff measure and Hausdorff dimension.
   - L^p-spaces, Young's inequality, Minkowski's inequality, Hölder inequality, Banach space, dual spaces, Jensen's inequality.

3. **Additional Topics**

   a) Homogeneous spaces, topologically equicontinuous measures, existence of invariant measures (Haar measures).

   b) Definition of weak derivatives, distributional derivatives, Sobolev space, regularization of functions in L^1_{loc} .

   Note: You will have a choice between problems on a) or b).

References:

C. Apostol, Mathematical Analysis

G. Folland, Real Analysis

H. Royden and P. Fitzpatrick, Real Analysis

W. Rudin, Real and Complex Analysis

E. Stein and R. Shakarchi, Princeton Lectures in Analysis III - Real Analysis.