2012 Topology Qualifying Exam Syllabus

I. *General Topology*

(1) topological spaces; bases and subbases; order topology; subspace topology; product topology; continuous functions and homeomorphisms; metric topology; open and closed maps; quotient topology.

(2) connectedness; connectedness of intervals in linear continua; intermediate value theorem; path-connectedness; local connectedness; local path connectedness; components; path components.

(3) compactness; extreme value theorem; tube lemma; compactness of $[a, b]$; Heine-Borel theorem; sequential and limit point compactness; Lebesgue number lemma; local compactness; compactification; Tychonoff's theorem.

(4) countability axioms; separation axioms; Urysohn's lemma; completely regular spaces and the evaluation map; Urysohn metrization theorem; Tietsze extension theorem.

(5) complete metric spaces; compactness in metric spaces; function spaces and the compact-open topology.

II. *Algebraic Topology*

(1) fundamental group; fundamental group functor; homotopy type; retractions; deformation retractions; simply connected and contractible spaces; $\pi_1(S^n)$; Brouwer and Borsuk–Ulam theorems in dimension two.

(2) free groups; free products of groups; group presentations; van Kampen theorem.

(3) topological manifolds; fundamental groups of surfaces; CW complexes with dimensions 1 and 2; classification of surfaces; Euler characteristic.

(4) covering space theory; unique path lifting property; lifting maps to covering spaces; equivalence of covering spaces; semi-local connectedness; universal covering space; existence of covering spaces; regular coverings and deck transformation groups.

(5) graphs and trees; fundamental groups of graphs; subgroups of free groups.

References:

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2001. Available online at 'www.math.cornell.edu/ hatcher/#ATI'.
- William Massey, *Algebraic Topology: An Introduction*, GTM Vol. 56, Springer-Verlag, 1990.
- James Munkres, *Topology (second edition)*, Prentice-Hall, 2000.

Additional Comments and Suggestions:

- The material in *Part I: General Topology* is encompassed by the following sections from Munkres:

  (1) Munkres: §§12–22 (chapter 2)

  (2) Munkres: §§23–25

  (3) Munkres: §§26–29, 37

  (4) Munkres: §§30–35

  (5) Munkres: §§43, 45, 46

- For *Part I: General Topology* it would also be recommended to study key examples of topological spaces such as those listed in the *Supplementary Exercises: Review of Basics* at the end of Chapter 4 of Munkres book (page 228). Problems 1–6 in that section provide good review problems for the material in Part I.

- Most of the material in *Part II: Algebraic Topology* is discussed in Chapter 1 and portions of Chapter 0 in Hatcher's book. It is also encompassed by the following sections of Massey and Munkres books:

  (1) Massey: chapter 2, Munkres: chapter 9.

  (2) Massey: chapters 3 and §§4.1–4.4, Munkres: chapter 11.

  (3) Massey: chapter 1 and §4.5, Munkres: chapter 12.

  (4) Massey: chapter 5, Munkres: chapter 13.

  (5) Massey: chapter 6, Munkres: chapter 14.

- The emphasis and level of detail of the various topics will be governed by the class lectures in the courses Math 5853/Fall 2011 and Math 5863/Spring 2012.