Algebra Qualifying Exam Syllabus

(Prepared by R. Schmidt, Spring 2013)

Disclaimer: The list of topics presented below is intended to be reasonably representative, but is not guaranteed to be exhaustive. All of these topics are covered in the course text "Algebra. Vol. 1: Fields and Galois Theory", by Falko Lorenz, Springer, 2006

1) **Groups**

   - Groups, subgroups, homomorphisms
   - Cosets and double cosets
   - Normal subgroups, kernels, quotients, the Fundamental Homomorphism Theorem, exact sequences
   - Cyclic groups, dihedral groups, symmetric groups, general linear groups over finite and infinite fields, and other standard examples
   - Classification of finitely generated abelian groups
   - Group actions, stabilizers, orbits, orbit formula, class equation
   - Solvable groups and p-groups
   - Sylow Theorems

2) **Rings**

   - Rings, subrings, homomorphisms, units
   - Ideals, kernels, quotients, the Fundamental Homomorphism Theorem, exact sequences
   - Prime ideals and maximal ideals
   - Integral domains, field of fractions
   - Irreducible elements and prime elements
   - Euclidean domains, PIDs, UFDs
   - Polynomial rings, factorization of polynomials, Gauss's Lemma, Eisenstein Criterion
   - $\mathbb{Z}/n\mathbb{Z}$, $(\mathbb{Z}/n\mathbb{Z})^\times$, Chinese Remainder Theorem, Euler φ-function
   - Tensor products of vector spaces and K-algebras, universal property

3) **Fields**

   - Fields, characteristic, standard examples, field extensions, degree
   - Finite subgroups of the multiplicative group of a field are cyclic
   - Transcendental elements, field of rational functions
   - Algebraic elements, minimal polynomial, algebraic closure
   - Normal extensions, normal closure, splitting fields
   - Separable polynomials, separable extensions, purely inseparable extensions, separability and inseparability degree
   - Galois theory, examples over $\mathbb{Q}$
   - Roots of unity, cyclotomic polynomials
   - Classification of finite fields, Frobenius automorphism