Syllabus for Ph.D. Qualifying Exam in Analysis
Spring 2013

A reference for these topics is the text *Real Analysis (4th edition)* by H.L. Royden and P. M. Fitzpatrick. Another useful reference is *Measure and Integral: An Introduction to Real Analysis* by R. L. Wheeden and A. Zygmund.

1. Topology of $\mathbf{R}^n$, convergence of sequences, continuity of functions, uniform continuity.

2. Lebesgue outer measure on $\mathbf{R}$, sets of measure zero, Lebesgue measurable sets and Lebesgue measure, nonmeasurable sets, the Cantor set and Cantor-Lebesgue function.

3. Lebesgue measurable functions, approximation by simple functions, Egorov's theorem, Lusin's theorem.

4. Riemann integral, Lebesgue integral, Chebyshev's inequality, Fatou's lemma, Monotone Convergence Theorem, Dominated Convergence Theorem, convergence in measure.

5. Vitali covering lemma, Lebesgue's theorem on derivatives of monotone functions, functions of bounded variation, Jordan decomposition theorem for functions, absolutely continuous and singular functions, Lebesgue decomposition theorem for functions, convex functions, Jensen's inequality.

6. L^p and l^p spaces, Young's inequality, Hölder's inequality, Minkowski's inequality, basics of metric spaces and Banach spaces, separability of L^p, Hilbert spaces, complete orthonormal bases for L^2, Bessel's inequality, Parseval's identity, Riesz-Fischer theorem on isometry between L^2 and l^2.

7. σ-algebras and measure spaces, signed measures (also called additive set functions), Jordan decomposition theorem and Hahn decomposition theorem for signed measures.

8. Integral with respect to a measure, absolutely continuous and singular set functions and measures, Radon-Nikodym theorem, Lebesgue decomposition theorem for measures, Riesz representation theorem for the dual of L^p.

9. Carathéodory-Hahn extension theorem, product measures, theorems of Fubini and Tonelli.