

Topology Qualifying Exam Syllabus

(Prepared by Ravi Shankar, Spring 2013)

Disclaimer: The list of topics presented below is intended to be reasonably representative, but is not guaranteed to be exhaustive. Most of these topics are covered in the course text “Topology”, by James Munkres, 2nd edition (Prentice Hall).

1) General/Point Set Topology

- Set theory and logic (including infinite sets and the Axiom of Choice)
- Topological spaces, continuous functions, product, metric and quotient topologies
- Connectedness, local connectedness, quasi-connectedness
- Compactness: local compactness, limit point compactness
- Countability and Separation axioms; the Urysohn Metrization theorem
- Smooth manifolds, Jacobian of a map between manifolds, examples of manifolds
- The Tychonoff theorem, paracompactness
- Complete metric spaces, Ascoli’s theorem

2) Algebraic Topology

- Fundamental group: homotopy of paths, covering spaces, $\pi_1(S^1, b_0)$
- Deformation retracts, homotopy equivalences, fundamental group of S^n
- Borsuk–Ulam theorem, Brouwer fixed point theorem (in dimension 2)
- Direct sums of abelian groups, Free groups
- The Seifert–van Kampen theorem, adjoining a 2-cell
- Fundamental groups of surfaces, first homology group of surfaces
- Cutting and pasting, finitely presented groups, the Classification theorem for surfaces
- Covering spaces: equivalence of covering spaces, the universal cover, regular cover
- Covering spaces: deck/covering transformations, existence of covering spaces
- Covering spaces of a graph, subgroups of a free group

3) Other Topics

- Group actions on topological spaces, orbits, stabilizers, homogeneous spaces
- Riemannian metrics on domains in the complex plane