

## Algebra Qualifying Exam Syllabus

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**Note:** The following list of topics is meant to be representative but not necessarily exhaustive. All of these topics are covered in the textbook *Abstract Algebra* by Dummit & Foote, 2nd edition.

### 1 Group theory

- Groups, subgroups, homomorphisms
- Cosets, double cosets, Lagrange's theorem
- Normal subgroups, kernels, quotients, Isomorphism theorems
- Group actions, orbits, stabilizers, Orbit–Stabilizer theorem, class equation
- Cyclic groups, dihedral groups, symmetric groups,  $p$ -groups, general and special linear groups, solvable groups
- Sylow theorems, semi-direct products, groups of small order
- Automorphism groups, permutation groups, simple groups

### 2 Rings

- Rings, sub-rings, homomorphisms, ideals
- Examples: polynomial rings, matrix rings, group rings, discrete valuation rings, local rings
- Quotient rings, Isomorphism theorems, Chinese Remainder theorem
- Prime ideals, maximal ideals, integral domains, division rings, fields, field of fractions

- Euclidean domains, PIDs, UFDs, irreducible elements, prime elements
- Polynomial rings, factorization of polynomials, Gauss' Lemma, Eisenstein's criterion

### 3 Fields

- Field extensions, algebraic extensions, degree of an extension
- Transcendental elements, algebraic elements, minimal polynomial, algebraic closure
- Straightedge and compass constructions, Fermat primes
- Splitting fields, normal extensions, automorphisms of fields, fixed fields
- Separable polynomials, separable extensions, purely inseparable extensions
- Galois extensions, Galois group of a Galois extension, Fundamental theorem of Galois theory
- Cyclotomic polynomials, abelian extensions
- Finite fields, classification of finite fields, Frobenius endomorphism
- Primitive Element theorem, roots of unity, composite extensions
- Solvable and radical extensions, Galois groups of cubics and quartics, unsolvability of the quintic
- Computation of Galois groups over  $\mathbb{Q}$