

Note: The following list of topics is meant to be representative but not necessarily exhaustive. All of these topics are covered in the textbook *Abstract Algebra* by David S. Dummit and Richard M. Foote, 2nd edition.

1 Groups

- Groups, subgroups, homomorphisms
- Cosets, Lagrange's theorem
- Normal subgroups, kernels, quotients, isomorphism theorems
- Group actions, orbits, stabilizers, orbit formula, class equation
- Cyclic groups, dihedral groups, symmetric groups, p -groups, nilpotent groups, solvable groups, free groups
- Sylow theorems, semi-direct products, groups of small orders
- Classification of finite abelian groups
- Automorphism groups, permutation groups, simple groups

2 Rings

- Rings, sub-rings, homomorphisms
- Ideals, kernels, quotient rings, Isomorphism theorems
- Prime ideals, maximal ideals
- Integral domains, field of fractions
- Euclidean domains, PIDs, UFDs
- Irreducible elements, prime elements, units
- Polynomial rings, factorization of polynomials, Gauss's Lemma, Eisenstein's criterion
- \mathbb{Z}_n , \mathbb{Z}_n^\times , Chinese Remainder theorem, Euler ϕ -function

3 Fields

- Fields, characteristic, standard examples, field extensions, degree
- Algebraic elements, minimal polynomial, algebraic closure
- Normal extensions, splitting fields
- Separable polynomials, separable and inseparable extensions
- Galois extensions, Galois group, Fundamental theorem of Galois theory
- Roots of unity, cyclotomic polynomials, abelian extensions
- Classification of finite fields, Frobenius automorphism
- Composite extensions, simple extensions
- Computation of Galois groups