SYLLABUS FOR THE PH.D. QUALIFYING EXAM IN ANALYSIS
AUGUST 2015, JANUARY 2016

The Ph.D. qualifying exam in analysis will cover the material you find in chapters 1–17, 19, 26 of [Bas13] and chapters 0–3, 5–7, 9 of [Fol99].

In the following table you can find a few central topics the exam will cover.

(1) Topology of $\mathbb{R}^n$, convergence of sequences, continuity of functions, uniform continuity.
(2) Cardinality, countable sets, uncountable sets, Schröder–Bernstein theorem, Zorn's lemma
(3) nonmeasurable sets in $\mathbb{R}^n$ and the "standard construction" of examples of such sets using Zorn's lemma (the Banach–Tarski paradox is also a good thing to have in mind)
(4) σ–algebras and measure spaces, monotone class theorem, Dynkin lemma (the functional version of the monotone class theorem), measurable functions and their approximation by simple functions, null sets and sets of measure zero
(5) outer measures, the Caratheodory lemma, the Caratheodory extension theorem − in particular how this methods can be used to construct the Lebesgue measure and Lebesgue–Stieltjes measures on $\mathbb{R}^n$
(6) Lebesgue integral, Monotone Convergence Theorem, Fatou's lemma, Dominated Convergence Theorem,
(7) Chebyshev–Markov inequality, convergence in measure, almost everywhere convergence, convergence in $\mathscr{L}^p$
(8) Riemann integral and its relation to the Lebesgue integral
(9) absolutely continuous and singular measures, signed measures, the Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition, the Lebesgue decomposition theorem
(10) product σ–algebra (and their generation), product measures, Tonelli's theorem, Fubini's theorem
(11) $\mathscr{L}^p$, L^p and ℓ^p spaces, Young's inequality, Jensen's inequality, Hölder's inequality, Minkowski's inequality, completeness of the above mentioned spaces,
(12) convolutions, Young's inequality for convolutions, differentiability of convolutions with respect to smooth functions (mollifiers), examples of $C_c^\infty(\mathbb{R}^n)$ functions, approximation of $\mathscr{L}^p(\mathbb{R}^n)$ by $C_c^\infty(\mathbb{R}^n)$ functions
(13) basics of metric spaces and Banach spaces, Hilbert spaces, orthonormal systems and orthonormal bases in Hilbert spaces, Bessel's inequality, Parseval's identity,
(14) bounded linear functionals and their norms, Riesz–representation type theorems for $C_c(X)$ (where X is a compact metric space), L^p, abstract Hilbert spaces
(15) absolutely continuous functions, functions of bounded variation, Lebesgue density points in $\mathbb{R}^n$, Lebesgue's differentiation theorem for increasing functions, for functions of bounded variation, for absolutely continuous functions, and for averages of locally integrable functions on $\mathbb{R}^n$
(16) Fourier transformation, Fourier coefficients, the corresponding inversion theorems, Plancherel theorem and Parseval's identity, Fourier transformation of functions of Schwartz class (the connection to tempered distributions)
(17) distributions, derivatives of distributions, convergence of distributions, the distribution naturally corresponding to (locally integrable) functions, convergence of (locally integrable) functions in the sense of distributions, the support of distributions, tempered distributions, Fourier transformation of tempered distributions

References

[Bas13] Richard F. Bass. Real Analysis for Graduate Students. 2 edition, 2013.
[Fol99] Gerald B. Folland. Real analysis. Pure and Applied Mathematics (New York). John Wiley & Sons, Inc., New York, second edition, 1999. Modern techniques and their applications, A Wiley-Interscience Publication.

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