Syllabus for Topology Qualifying Exam, 2014

The 2013–2014 topology graduate course used the books *Topology* (second edition) by Munkres and *Algebraic Topology* by Hatcher (chapters 0 and 1). The book *Algebraic Topology: An Introduction* by Massey is also recommended, as it provides more detail than Hatcher in some areas.

General Topology:

I. Topological spaces and continuous maps (Munkres, sections 12–20, 22)
1. Topological spaces, bases, subbases
2. The order topology
3. The product topology (two factors)
4. The subspace topology
5. Closed sets and limit points, Hausdorff spaces
6. Continuous maps, homeomorphisms, local continuity, pasting lemma, maps into products
7. The product topology (general case), **maps into product spaces**, box topology
8. Metric spaces, uniform topology
9. The quotient topology, maps out of quotient spaces

II. Connectedness and compactness (Munkres, sections 23–27, 29)
1. Connected spaces, connectedness of products
2. Connectedness in linear continua, intermediate value theorem, path connectedness
3. Components and local connectedness
4. Compact spaces: continuous maps, products, tube lemma, finite intersection property
5. Extreme value theorem, Lebesgue number lemma
6. local compactness, one-point compactification
7. Compactness of $I = [0, 1]$ is compact (using only the least-upperbound-property of $\mathbb{R}$)

III. Countability and separation axioms (Munkres, sections 30–32)
1. First and second countability axioms
2. regular spaces, normal spaces

IV. Urysohn's Lemma and applications (Munkres, sections 33, 35, 36)
1. Urysohn's Lemma, separation by continuous functions **(Construction of Urysohn function)**
2. Tietze extension theorem
3. Partitions of unity
4. Embeddings of manifolds

V. Function spaces (Munkres, sections 43, 45–46)

1. uniform metric
2. topology of pointwise convergence (= point-open topology)
3. compact-open topology

V. Other topics (Munkres, sections 37, 46)

1. The Tychonoff theorem (statement and applications only)
2. The compact-open topology, the evaluation map, induced maps

Algebraic Topology:

VI. The fundamental group (Munkres, sections 51–52, 54–55, 57–60; Hatcher, section 1.1)

1. Path homotopy, properties
2. Fundamental group, induced homomorphisms
3. Fundamental group of the circle (via a covering space)
4. Retractions and the fundamental group, Brouwer fixed point theorem
5. The Borsuk-Ulam theorem, applications
6. Deformation retractions, homotopy equivalences

VII. Covering spaces (Hatcher, section 1.3; Munkres, sections 53–54, 79–82)

1. Definition of covering spaces
2. Path lifting and uniqueness
3. Injectivity of induced map on fundamental group
4. The lifting criterion, uniqueness of lifts
5. Construction of covering space (For a subgroup H of $\pi_1(B, b_0)$, a covering space $p : (E, e_0) \to (B, b_0)$ such that $p_*(\pi_1(E, e_0)) = H$; Describe the space E, its topology and the map p, and an evenly covered neighborhood of any $b \in B$).
6. Equivalence of covering spaces, correspondence between subgroups and covering spaces (see also Massey, chapter 5, sections 3–6)
7. Covering transformations, regular covering spaces

VIII. The van Kampen theorem (Hatcher, section 1.2 and chap. 0; Munkres, sections 69–73)

1. Free products of groups, existence, the mapping property
2. The van Kampen theorem
3. 1- and 2-dimensional cell complexes, attaching cells, collapsing a contractible subcomplex
4. Generators and relations, fundamental groups of cell complexes

IX. Group actions (Hatcher, section 1.3)

1. Properly discontinuous action
2. Orbit spaces

IX. Graphs and free groups (Hatcher, section 1.A; Massey, chapter 6)

1. Graphs = 1-dimensional cell complexes
2. Cayley graph, Cayley complex of a group