# ALGEBRA QUALIFYING EXAM SYLLABUS FOR 2017

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**Note**: The following list of topics is meant to be representative but not necessarily exhaustive. All of these topics are covered in the textbook Abstract Algebra by David S. Dummit and Richard M. Foote, 2nd edition.

## Groups.

- Groups, subgroups, homomorphisms
- Cosets, Lagranges theorem
- Normal subgroups, kernels, quotients, isomorphism theorems
- Group actions, orbits, stabilizers, orbit formula, class equation
- Cyclic groups, dihedral groups, symmetric groups, p-groups, nilpotent groups, solvable groups, free groups, presenting groups by generators and relations
- Sylow theorems, direct and semi-direct products, groups of small orders
- Classification of finite abelian groups
- Automorphism groups, permutation groups, simple groups, composition series and composition factors

## Rings and Algebras.

- Rings and associative algebras, sub-rings and sub-algebras, homomorphisms, graded rings and algebras
- Ideals, kernels, quotient rings, Isomorphism theorems
- Prime ideals, maximal ideals
- Multiplicatively closed sets, localization
- Irreducible elements, prime elements, units
- Integral domains, Euclidean domains, PIDs, UFDs
- Polynomial rings, factorization of polynomials, Gauss's Lemma, Eisenstein's criterion
- Comaximal ideals and the Chinese Remainder theorem

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**Modules.**

- Left/right/bi modules, submodules, graded modules for graded rings and modules, direct sums, homomorphisms
- Quotient Modules, Isomorphism theorems
- Free modules, projective modules
- Tensor products of vector spaces and bimodules
- Classification of finitely generated modules over a PID
- Modules for a polynomial ring, group algebra of a group, and other standard examples.

**Fields and Galois Theory.**

- Fields, characteristic, standard examples, field extensions, degree
- Algebraic elements, minimal polynomial, algebraic closure
- Normal extensions, splitting fields
- Separable polynomials, separable and inseparable extensions
- Galois extensions, Galois group, Fundamental theorem of Galois theory
- Roots of unity, cyclotomic polynomials, abelian extensions
- Classification of finite fields, Frobenius automorphism
- Composite extensions, simple extensions
- Fundamental Theorem of Galois Theory, computation of Galois groups and intermediate fields

**Category Theory.**

- Categories and functors, standard examples
- Universal property of various standard constructions (quotient groups/rings/modules, free groups/modules, localizations, direct products, etc.)
- Natural transformations