Syllabus for Ph.D. Qualifying Exam in Analysis
2017

The text used for the qualifying courses in 2016–2017 was *Real Analysis for Graduate Students* by Richard F. Bass. Other useful references for this material are *Measure and Integral: An Introduction to Real Analysis* by R. L. Wheeden and A. Zygmund, and *Real Analysis* by H.L. Royden and P. M. Fitzpatrick.

1. Metric spaces, the topology of metric spaces: open and closed sets, compact sets, convergent sequences, Cauchy sequences, completeness. Continuous functions. The difference between pointwise and uniform convergence of a sequence of functions.

2. Algebras and σ-algebras of sets, measures, outer measures. Borel σ-algebra. Lebesgue-Stieltjes measures on $\mathbb{R}$. Lebesgue measure on $\mathbb{R}$. Lebesgue measurable sets. Nonmeasurable sets. The Caratheodory extension theorem.

3. Measurable functions, approximation of measurable functions by simple functions, Lusin's theorem, Egorov's theorem.

4. Lebesgue integral with respect to a measure, and its properties. Chebyshev's inequality, Fatou's lemma, Monotone Convergence Theorem, Dominated Convergence Theorem. Riemann integral and its relation to the Lebesgue integral. Convergence in measure, the relation between convergence in measure and pointwise convergence.

5. Product measures. The Fubini-Tonelli theorem.

6. Signed measures. Mutually singular measures. The Jordan and Hahn decomposition theorems. Absolutely continuous measures. Radon-Nikodym theorem, Lebesgue decomposition theorem.

7. Vitali covering lemma, Hardy-Littlewood maximal function, Lebesgue's theorem on derivatives of indefinite integrals. Functions of bounded variation, Lebesgue's theorem on derivatives of monotone functions, absolutely continuous functions.

8. L^p and l^p spaces, Young's inequality, Hölder's inequality, Minkowski's inequality. Completeness of L^p. Convolutions. Separability of L^p. L^p duality.

9. Hilbert spaces and their basic properties, the representation theorem for bounded linear functionals on Hilbert space as inner products. Orthonormal sets, Fourier expansion of an element of a Hilbert space with respect to an orthonormal set. Bessel's inequality, complete orthonormal sets, Parseval's identity.