Syllabus for Topology Qualifying Exam, 2017

The 2016-2017 topology graduate course and qualifying exam used/are using as reference the books Topology (second edition) by Munkres and Algebraic Topology by Hatcher (chapters 0 and 1).

General Topology

I. Topological spaces and continuous maps

1. Topological spaces, bases, subbases
2. Special topologies: order topology, product topology, box topology, subspace topology, quotient topology and maps out of quotient spaces
3. Closed sets and limit points, Hausdorff spaces, T1, etc.
4. Continuous maps, homeomorphisms, local continuity, pasting lemma, maps into products
5. Metric spaces, uniform topology

II. Connectedness and compactness

1. Connected spaces, connectedness of products, connectedness in linear continua, intermediate value theorem, path connectedness
2. Components and local connectedness
3. Compact spaces: continuous maps, products, tube lemma, finite intersection property
4. Extreme value theorem, Lebesgue number lemma
5. local compactness, limit-point compactness, one-point compactification
6. Compactness of $I = [0, 1]$ is compact (using only the least-upper bound property of $\mathbb{R}$)

III. Countability and separation axioms

1. First and second countability axioms
2. regular spaces, normal spaces

IV. Urysohns Lemma and applications

1. Urysohns Lemma, separation by continuous functions (Construction of Urysohn function)
2. Tietze extension theorem
3. Partitions of unity
4. Embeddings of manifolds

V. Tychonoff theorem (statement and applications only)

VI. Complete metric spaces and Function spaces

1. uniform metric
2. space-filling curve
3. topology of pointwise convergence (= point-open topology)
4. The compact-open topology, the evaluation map, induced maps

Algebraic Topology

I. The fundamental group

1. Path homotopy, properties
2. Fundamental group, induced homomorphisms
3. Retractions and the fundamental group, Brouwer fixed point theorem
4. The Borsuk-Ulam theorem, applications
5. Deformation retractions, homotopy equivalences

II. Covering spaces

1. Lifting: paths and in general
2. Injectivity of induced map on fundamental group
3. Construction of covering space (For a subgroup H of $\pi_1(B, b_0)$, a covering space $p\colon (E, e_0) \to (B, b_0)$ such that $p_*(\pi_1(E, e_0)) = H$; Describe the space E, its topology and the map p, and an evenly covered neighborhood of any $b \in B$).
4. Equivalence of covering spaces, correspondence between subgroups and covering spaces
5. Covering transformations, regular covering spaces

III. The van Kampen theorem

1. Free products of groups, existence, the mapping property
2. The van Kampen theorem
3. 1- and 2-dimensional cell complexes, attaching cells, collapsing a contractible subcomplex
4. Generators and relations, fundamental groups of cell complexes

IV. Graphs and free groups

1. Graphs = 1-dimensional cell complexes
2. Cayley graph, Cayley complex of a group, presentation complex of a group

Other

I. Group Actions on topological spaces

1. Orbits, stabilizers, homogeneous spaces, orbit spaces
2. Properly discontinuous action