

Algebra Qualifying Exam Syllabus

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Note: The following list of topics is meant to be representative but not necessarily exhaustive. All of these topics are covered in the textbook *Abstract Algebra* by David S. Dummit and Richard M. Foote.

Group Theory

- Groups, subgroups, cosets, quotients, homomorphisms, isomorphism theorems
- Free groups, presentations, generators and relations
- Finitely generated abelian groups, finite groups, cyclic groups, permutation groups, dihedral groups, p -groups
- Solvable and nilpotent groups, normal series
- Group actions, stabilizers, orbits, class equation
- Sylow theorems

Ring Theory

- Rings, ideals, quotients, homomorphisms, isomorphism theorems
- Maximal ideals, prime ideals, radical
- Integral domains, euclidean rings, principal ideal domains, unique factorization domains, polynomial rings, field of fractions, Gauss lemma, Eisenstein criterion
- Modules, free modules, quotient modules, tensor product of modules

Field Theory

- Fields, algebraic extensions, minimal polynomial, degree, algebraic closure
- Splitting fields, separable polynomials, primitive element theorem
- Transcendental elements and extensions, function fields
- Finite fields
- Norm and trace
- Normal extensions, Galois theory