SYLLABUS FOR PH.D. QUALIFYING EXAM IN ANALYSIS 2018

The real analysis qualifying exam will be based on material related to the topics listed below. This list is not meant to be exhaustive, but is intended to be a reasonably representative guide to the subjects that will be covered in the exam.

(a) **Lebesgue measure:** Lebesgue outer measure, Lebesgue measurable sets, Approximation of measurable sets, Nonmeasurable sets.

(b) **Lebesgue integration:** Measurable functions, Simple approximation, Egoroff and Lusin Theorems, Integration of measurable functions, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem, Uniform integrability and tightness, Convergence in measure, Comparison with Riemann Integral, Characterizations of Lebesgue and Riemann integrability.

(c) **Differentiation and Integration:** Continuity and differentiability of monotone functions, Functions of bounded variation, Absolutely continuous functions, Derivatives of indefinite integrals, Convex functions.

(d) **Metric spaces:** Metrics, Open and closed sets, Completeness, Compactness, Separability, Continuous and uniformly continuous mappings, Lebesgue Covering Lemma, Arzèla-Ascoli Theorem, Baire Category Theorem, Banach Contraction Principle.

(e) **Topological spaces:** Topologies, Bases, Separability, Compactness, Connectedness, Continuous mappings, Initial topology induced by a family of mappings, Nets, Urysohn's Lemma, Tietze Extension Theorem, Tychonoff Product Theorem, Stone-Weierstrass Theorem.

(f) **Normed and Banach spaces:** Norms, Bounded linear operators, Open Mapping and Closed Graph Theorems, Uniform Boundedness Principle, Weak and weak* topologies, Hahn-Banach Theorem, Reflexive spaces, Alaouglu's Theorem.

(g) **Hilbert spaces:** Inner product, Orthogonality, Duality (Riesz representation), Bessel's inequality, Orthonormal bases, Adjoints and symmetry for linear operators,

(h) **General measure spaces:** Measures and measurable sets, Signed measures, Hahn and Jordan decompositions, Outer measures, Carathéodory extensions, Product measures.

(i) **Integration over general measure spaces:** Measurable functions, Simple approximation, Egoroff's Theorem, Integration of measurable functions with respect to a general measure, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem, Radon-Nikodym Theorem, Fubini and Tonelli Theorems, Lebesgue measure on $\mathbb{R}^n$, Lebesgue-Stieltjes measures.

(j) L^p **spaces:** Hölder and Minkowski inequalities, Completeness and separability of L^p, Dense subspaces of L^p (e.g. simple functions, continuous functions with compact support, etc.), Duality (Riesz Representation), Weak sequential convergence and compactness.

The textbook used for MATH 5453-5463 during the 2017-2018 academic year was *Real Analysis* by H.L. Royden and P. M. Fitzpatrick. Other useful references are:

- *Real Analysis for Graduate Students*, by Richard Bass (available online for free).
- *Real Analysis*, by Gerald B Folland.
- *Analysis*, by Elliot H. Lieb and Michael Loss.