Syllabus for Qualifying Exam, 2018

The 2017-2018 topology graduate course and qualifying exam used as reference the books Topology (Second edition) by Munkres, Sections 2.2, 2.4, 2.6, 2.7 of Introduction to Topology by Gamelin and Greene, and some early sections of Algebraic Topology by Hatcher as indicated below.

General Topology

1. Metric spaces: metric topology, sequences, limits, compactness in metric spaces (including various characterizations), completeness, Lipschitz, biLipschitz
2. Basic definitions/operations: subspace topology, basis, subbasis, continuous functions, homeomorphisms, pasting lemma, closed sets and adherents, closed map, open map
3. Products: box and product topology on finite and infinite products, Tychonoff's Theorem (statement and applications), continuous maps into products
4. Separation Axioms: Hausdorff, normal, regular, Urysohn Lemma, Urysohn Metrization Theorem
5. Connectedness, Path-connectedness, connectedness of products, connected components, path-components
6. Compactness: various definitions and kinds (compact, sequentially compact, limit point compact), images under continuous maps, Lebesgue number, tube lemma, compact sets in $\mathbb{R}^n$
7. Quotient topology, maps from quotient spaces
8. 2nd countable, Baire space, Baire category theorem
9. Compact-open topology on spaces of functions
10. Space-filling curve
11. Brouwer fixed point theorem in dimension 2

Algebraic Topology

1. Hatcher, Chapter 0. Homotopy and homotopy type. CW complexes. Operations on spaces & CW complexes. Homotopy extension property (HEP). Homotopy lifting property (HLP). Retractions and deformation retractions.

2. Hatcher, Chapter 1.1. Paths and homotopies of paths, definition and examples of fundamental group, induced homomorphisms (functoriality) and applications.

3. Hatcher, Chapter 1.2. van Kampen's theorem and examples, fundamental group of CW complex.

4. Hatcher, Chapter 1.3. Definition and basic properties of covering spaces (e.g., lifting criterion & HLP), classification of covering spaces (i.e., injectivity on π_1 and the Galois correspondence), regular/normal covers including examples, irregular covers including examples, group actions and deck transformations.