ALGEBRA QUALIFYING EXAM SYLLABUS FOR 2019

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The following list of topics is meant to be representative but not necessarily exhaustive. All of these topics are covered in the textbook *Abstract Algebra* by David S. Dummit and Richard M. Foote, 3rd edition.

(1) Groups

- Groups, subgroups, homomorphisms
- Cosets, Lagrange's Theorem
- Normal subgroups, kernels, quotients, isomorphism theorems
- Group actions, orbits, stabilizers, centralizers, normalizers, class equation
- Cyclic groups, dihedral groups, symmetric groups, alternating groups, matrix groups
- Free groups, presenting groups by generators and relations
- Direct products, semi-direct products, automorphism groups, composition series
- Sylow Theorems, groups of small order
- Classification of finite abelian groups

(2) Rings

- Rings, subrings, homomorphisms
- Ideals, kernels, quotient rings, isomorphism theorems
- Prime ideals, maximal ideals
- Integral domains, rings of fractions
- Chinese Remainder Theorem, Euler φ-function, $(\mathbb{Z}/n\mathbb{Z})^\times$
- Euclidean domains
- Polynomial rings, polynomial division algorithm
- Principal ideal domains (PIDs), Noetherian rings, Hilbert Basis Theorem
- Irreducible elements, Unique Factorization Domains (UFDs), Gauss' Lemma

(3) Modules

- Modules, homomorphisms of modules, categories of modules
- Modules of a polynomial ring, relation with matrices
- Direct sums of modules, extensions of modules
- Tensor products
- Classification of modules of PIDs. Rational and Jordan canonical forms

(4) Fields

- Fields, homomorphisms of fields, standard examples, characteristic of a field
- Field extensions, degree of an extension
- Simple extensions, algebraic extensions, splitting fields, algebraic closure
- Separable extensions, Galois extensions, characterizations of Galois extensions
- Galois correspondence, i.e. the Fundamental Theorem of Galois Theory
- Extensions of finite fields, the Frobenius endomorphism