Real Analysis   Qualifying Examinations
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Reference books: *Real Analysis* by H. Royden, *Real Analysis* by Stein and Shakarchi

Content

1. Preparation: Set theory (Algebras of sets, countable and uncountable sets). Real number system (open sets, continuous functions, uniformly continuous and compactness properties).

2. Lebesgue measure: Outer measure, Lebesgue measure and measurable sets, properties of measurable sets, measurable functions and properties, Littlewood's principles, example of nonmeasurable sets.

3. Lebesgue integral: Building up Lebesgue integral theory for various measurable functions, the corresponding convergence theorems. Pointwise convergence, uniform convergence and convergence in measure. Application of Lebesgue integral to the theory of Riemann integral.

4. Differentiation and integration: Vitali covering lemma, differentiability of monotone function, functions of bounded variation, absolutely continuous functions, relation between differentiation and integration; Properties of convex functions, Jensen inequality.

5. Metric spaces: Open sets, continuous and uniformly continuous functions, homeomorphisms and isometries, completeness and compactness, Borel-Lebesgue Theorem, Ascoli-Arzelá Theorem.

6. General measure and integration: General measure space, measurable functions, building up general integral theory (the difference between Lebesgue integral and general integral), convergence theorems, signed measure and Hahn decomposition, general integral via signed measure, definition of product measure, Fubini's Theorem, Tonelli's Theorem.

7. L^p spaces: Definition of L^p spaces, Minkowski and Hölder inequalities, completeness and approximation of L^p spaces, embedding in L^p spaces, Riesz representation theorem, Radon-Nikodym Theorem.

8. Introduction of Banach spaces: Linear operator, Hilbert spaces, Riesz representation Theorem in Hilbert spaces.